

ONE- AND TWO-NEUTRON CAPTURE REACTIONS OF LIGHT NUCLEI IN NUCLEAR ASTROPHYSICS

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We discuss models to calculate one- and two-neutron capture reactions on light nuclei. These are applied to calculate the reaction rates of $^{15}\text{N}(n,\gamma)^{16}\text{N}$, $^{16}\text{N}(n,\gamma)^{17}\text{N}$ and $^4\text{He}(2n,\gamma)^6\text{He}$. The possible astrophysical importance is discussed.

1 Introduction

Neutron capture reactions on light nuclei play a role in various astrophysical scenarios. In the framework of Inhomogeneous Big Bang Models a high neutron flux can bridge the mass 5 and mass 8 gaps. Subsequent neutron capture reactions may trigger a primordial r-process¹. Another site for neutron capture reactions is the high-entropy bubble formed during a type II supernova^{2,3,4}. Due to the photodisintegration at very high temperatures an α -rich environment is created. When the temperature has dropped heavier elements can be built up mainly by α - and neutron-capture reactions. Again a critical question is how the mass 5 and mass 8 gaps can be bridged.

In Section 2 we describe our model to calculate one-neutron capture reactions. This model is applied to the reactions $^{15}\text{N}(n,\gamma)^{16}\text{N}$ and $^{16}\text{N}(n,\gamma)^{17}\text{N}$. In Section 3 we will describe the theory of calculating a two-neutron capture reaction in a three-body model. We calculate the reaction rate of $^4\text{He}(2n,\gamma)^6\text{He}$ and compare the result with other works. In the final chapter we discuss and summarize our results.

2 One-Neutron Capture Reactions

One-nucleon capture reactions on light nuclei are usually dominated by direct capture (DC) to bound states and resonant capture to single, isolated resonances above the threshold. The level density above the threshold is normally low, i.e. between 0–10 levels per MeV. We calculate the cross section and reaction rate of the capture reaction with a hybrid model. The DC cross section is evaluated in a potential model. For the resonances we use the Breit–Wigner formula. We will discuss both contributions separately.

2.1 Direct Capture

The potential model is described by different authors^{5,6,7}. We use real folding potentials as optical potentials^{6,8}.

The DC cross section of a transition to a bound state is determined by the overlap of the scattering wave function, the bound-state wave function and the electromagnetic transition-operator. In most cases only E1-transitions need to be taken into account.

The total nonresonant cross section $\sigma_{\text{tot}}^{\text{DC}}$ is determined by the direct capture transitions σ_i^{DC} to all bound states multiplied with the single particle spectroscopic factors C^2S_i :

$$\sigma_{\text{tot}}^{\text{DC}} = \sum_i (C^2S)_i \sigma_i^{\text{DC}} \quad . \quad (1)$$

The spectroscopic factors can be determined experimentally from other reactions, e.g., the spectroscopic factor of a (n, γ)-reaction can be obtained from the (d,p)-reaction. Alternatively the spectroscopic factors can also be determined from shell-model calculations.

From the direct capture cross section we can obtain the reaction rate by integrating over a Maxwell–Boltzmann velocity distribution. We parametrize the non-resonant contribution to the rate

as

$$N_A < \sigma v >_{\text{nr}} = A + BT_9 - CT_9^D \quad \text{cm}^3 \text{mole}^{-1} \text{s}^{-1} \quad , \quad (2)$$

where T_9 denotes the temperature in GK. The constant term represents the s-wave capture contribution and the term proportional to T_9 the p-wave capture. All other contributions, i.e. higher partial waves and deviations from the conventional behaviours, are fitted in the third term.

2.2 Single, Isolated Resonances

The cross section of a single isolated resonance in neutron capture M processes is well described by the Breit–Wigner formula^{9,10}:

$$\sigma_r(E) = \frac{\pi \hbar^2}{2\mu E} \frac{(2J+1)}{2(2j_t+1)} \frac{\Gamma_n \Gamma_\gamma}{(E_r - E)^2 + \left(\frac{\Gamma_{\text{tot}}}{2}\right)^2} \quad , \quad (3)$$

where J and j_t are the spins of the resonance level and the target nucleus, respectively, E_r is the resonance energy. The partial widths of the entrance and exit channels are Γ_n and Γ_γ , respectively. The total width Γ_{tot} is the sum over the partial widths of all channels. The neutron partial width Γ_n can be expressed in terms of the single-particle spectroscopic factor C^2S and the single-particle width $\Gamma_{\text{s.p.}}$ of the resonance state^{11,12}

$$\Gamma_n = (C^2S) \Gamma_{\text{s.p.}} \quad . \quad (4)$$

The single-particle width $\Gamma_{\text{s.p.}}$ can be calculated from the scattering phase shifts of a scattering potential with the potential depth being determined by matching the resonance energy.

The gamma partial widths Γ_γ are calculated from the electromagnetic reduced transition probabilities $B(J_i \rightarrow J_f; L)$ which carry the nuclear structure information of the resonance states and the final bound states¹³. The reduced transition rates are computed within the framework of the shell-model.

The resonant reaction rate for an isolated narrow resonance can be expressed as^{11,12}

$$N_A < \sigma v >_r = 1.54 \times 10^5 \mu^{-3/2} T_9^{-3/2} \times \sum_i (\omega \gamma)_i \exp(-11.605 E_i / T_9) \text{ cm}^3 \text{ mole}^{-1} \text{ s}^{-1}, \quad (5)$$

where the resonance strength $\omega \gamma$ is defined as

$$\omega \gamma = \frac{2J + 1}{2(2j_t + 1)} \frac{\Gamma_n \Gamma_\gamma}{\Gamma_{\text{tot}}} \quad (6)$$

and has to be inserted in eV in Eq. 5. The resonance strength can be determined experimentally or derived from the calculated partial widths.

2.3 One-Neutron Capture on N-Isotopes

We will now consider one-neutron capture reactions on neutron-rich N-isotopes. We start with the reaction $^{15}\text{N}(n, \gamma)^{16}\text{N}$ which is known experimentally at stellar energies¹⁴. Therefore this reaction can be used as benchmark test for the validity of our model.

The considered transitions for the direct capture of this reaction are listed in Table 1. The four lowest states of ^{16}N can be described by a coupling of the $1/2^-$ ground state of ^{15}N with a neutron from the $1d_{5/2^-}$ (resulting in the 2^- ground state and a 3^- excited state) and the $2s_{1/2^-}$ -subshell (resulting in the low-lying excited states 0^- and 1^-). In a shell-model description these states are good one-particle states. Therefore the spectroscopic factors should be close to unity. In fact the shell-model calculations result in spectroscopic factors of about 0.9 for all states¹⁴. However, the spectroscopic factors from a (d,p)-experiment are considerably lower by an average factor of around 1.8¹⁵. The reason for this discrepancy is not known. In our DC calculations we use the experimental spectroscopic factors.

Table 1: Considered transitions for the direct capture reactions on N-isotopes. Transitions with very small contributions are not included in the table. The Q-values are in MeV.

reaction	Q-value	J^π	E_x (MeV)	transition	C^2S
$^{15}\text{N}(n,\gamma)^{16}\text{N}$	2.491	2^-	0.000	$p \rightarrow 1d_{5/2}$	0.550
		0^-	0.120	$p \rightarrow 2s_{1/2}$	0.460
		3^-	0.298	$p \rightarrow 1d_{5/2}$	0.540
		1^-	0.397	$p \rightarrow 2s_{1/2}$	0.520
$^{16}\text{N}(n,\gamma)^{17}\text{N}$	5.883	$1/2^-$	0.000	$p \rightarrow 1d_{5/2}$	0.589
		$5/2^-$	1.907	$p \rightarrow 1d_{5/2}$	0.207
		$7/2^-$	3.129	$p \rightarrow 1d_{5/2}$	1.457
		$5/2^-$	4.415	$p \rightarrow 2s_{1/2}$	0.921

All transitions listed in Table 1 result from an incoming p-wave. The s-wave contribution which dominates at very low energies can be obtained directly by extrapolating the thermal absorption cross section with the $1/v$ -law.

The resonance parameters are listed in Table 2. The total width of the 862 keV resonance of $\Gamma = 15 \text{ keV}$ ¹⁶ corresponds to the neutron width of the state. The γ width was estimated with the recommended upper limit as $\Gamma_\gamma = 4.2 \text{ eV}$ ¹⁴. From the shell model calculation we obtain a width $\Gamma_\gamma = 0.455 \text{ eV}$ which reduces the resonance strength by about one order of magnitude. However, this resonance only has a small influence on the reaction rate at very high temperatures. Therefore this change of the resonance strength barely changes the reaction rate.

The parameters of the direct capture reaction rate (see Eq. 2) are listed in Table 3. The resulting reaction rate agrees very well with the rate of Meissner *et al.*¹⁴ The rate is clearly dominated by the direct capture contribution. Both rates show good agreement with the experimental data. For a discussion of the experimental data we refer the reader to the paper of Meissner¹⁴.

The bound levels of ^{17}N are known from experiment. We used

Table 2: Adopted values for the resonance parameters for capture reactions on nitrogen isotopes.

reaction	E_x [MeV]	J^π	E_{res} [MeV]	Γ_n [eV]	Γ_γ [eV]	$\omega\gamma$ [eV]
$^{15}\text{N}(n,\gamma)^{16}\text{N}$	3.360	1^+	0.862	15 000	0.455	0.341
$^{16}\text{N}(n,\gamma)^{17}\text{N}$	5.904	$7/2^-$	0.021	0.032	$4.80 \cdot 10^{-2}$	0.015
	6.121	$5/2^+$	0.238	1.2	$4.80 \cdot 10^{-2}$	0.027
	6.325	$3/2^+$	0.442	20	$5.46 \cdot 10^{-2}$	0.022
	6.372	$7/2^+$	0.489	20	$1.52 \cdot 10^{-2}$	0.012
	6.373	$5/2^+$	0.490	600	0.110	0.066
	6.470	$1/2^+$	0.587	1 750	2.510	0.501
	6.685	$3/2^-$	0.802	12 500	5.660	2.263
	6.737	$7/2^+$	0.854	70	$4.17 \cdot 10^{-2}$	0.033
	6.835	$3/2^+$	0.952	360	0.478	0.191

Table 3: Parameters for the direct-capture contribution to the reaction rate.

	A	B	C	D
$^{15}\text{N}(n,\gamma)^{16}\text{N}$	3.18	3783.4	335.2	1.716
$^{16}\text{N}(n,\gamma)^{17}\text{N}$		3649.9	437.5	1.633

spectroscopic factors calculated in the shell-model for the DC calculation. The important DC transitions are listed in Table 1. Several levels are known above the threshold but without spin/parity assignments. It is impossible to assign the levels with the help of our shell model calculation. Therefore we use the shell-model energies for the resonances. The adopted resonance parameters are shown in Table 2. The two resonances at 21 keV and 238 keV are the main contributions to the reaction rate.

Since there is no s-wave transition the parameter A of the reaction rate vanishes (see Table 3). We find a considerable enhancement of our rate compared to the calculation of Rauscher *et al.*¹ In that work the resonance at 197 keV was given a hypothetical $5/2^+$ assignment. Since no resonance at lower energies was included,

our 21 keV resonance causes a strong increase of the rate at low temperatures.

3 Two-Neutron Capture Reactions

In this section we will discuss the possibilities to calculate a three-body reaction rate. We will apply the theory to the reaction ${}^4\text{He}(2n, \gamma){}^6\text{He}$.

The reaction rate of ${}^4\text{He}(2n, \gamma){}^6\text{He}$ can be calculated as a sequential two-step process. We call this method the ${}^6\text{He} \equiv \alpha+n+n$ approach. Here it is assumed that in a first step the unstable nucleus ${}^5\text{He}$ is formed via the reaction ${}^4\text{He} + n \rightarrow {}^5\text{He}$ (negative Q-value Q_1). The second step ${}^5\text{He} + n \rightarrow {}^6\text{He} + \gamma$ (positive Q-value Q_2) is treated as a two-body problem in the initial and as a three-body problem in the final state. The values for spin-parities, Q-values and widths used in our calculations are given in Table 4. The total Q-value Q_{12} is the sum of the Q-values Q_1 and Q_2 .

Table 4: Spin/parities, Q-values and widths (in MeV) used in our $\alpha+n+n$ calculations of the reaction rate for the reaction ${}^4\text{He}(2n, \gamma){}^6\text{He}$.

$J^\pi({}^4\text{He})$	$J^\pi({}^5\text{He})$	$J^\pi({}^6\text{He})$	Q_1	Q_2	Q_{12}	Γ_c
0^+	$3/2^-$	0^+	-0.89	1.87	0.98	0.76

In this two-step model the reaction rate of ${}^4\text{He}(2n, \gamma){}^6\text{He}$ is given by

$$N_{\text{He}}^2 \langle 2n^4\text{He} \rangle = 2 \int \int dE_1 dE_2 \frac{\hbar}{\Gamma(E_1)} \frac{d < n^4\text{He} >}{dE_1} \frac{d < n^5\text{He} >}{dE_2} . \quad (7)$$

The energies E_1 and E_2 denote the center-of-mass collision energies of the first (second) neutron with the ${}^4\text{He}$ (${}^5\text{He}$) nucleus. The width of the intermediate state, $\Gamma(E_1)$, is energy-dependent. The differential rates of the two steps represent the product of the cross

section with the Maxwell–Boltzmann distribution:

$$\frac{d \langle \sigma v \rangle}{dE} = \left(\frac{8}{\pi \mu} \right) \left(\frac{1}{kT} \right)^{3/2} \sigma(E) E \exp \left(-\frac{E}{kT} \right) . \quad (8)$$

The whole procedure of calculating the cross section is described in detail by Efros *et al*¹⁷ and by Balogh¹⁸.

The numerical results for the reaction rate $N_A^2 \langle 2n^4\text{He} \rangle$ calculated in this model are fitted to the expression

$$N_A^2 \langle 2n^4\text{He} \rangle = \sum_i a_i T_9^i \text{ cm}^6 \text{ s}^{-1} \text{ mole}^{-2} \quad (9)$$

in the temperature regions $0.1 \leq T_9 < 1$ and $1 \leq T_9 \leq 3$. From a comparison with the situation of small resonances it should be possible to perform a fit to a function $\sim a_1 T_9^{a_2} \exp(-a_3/T_9)$, but it turned out that such a fit leads only for $T_9 \geq 1$ to useful results. Therefore, we decided to use the parametrization of Eq. (9), which reproduces the calculated data with an error less than 0.25% in the whole temperature range. The parameters are given in Table 5.

In the work of Danilin *et al*¹⁹ the wave functions for ground and scattering states of the halo nucleus ${}^6\text{He}$ are calculated in an $\alpha+n+n$ three-body model and are used to predict the strengths of nuclear and electric dipole excitations. From the astrophysical point of view we are especially interested in the electric dipole strength function dB^{E1}/dE_γ , which is directly connected to the cross section for photodisintegration $\sigma_\gamma^{\text{E1}}$ of the nucleus ${}^6\text{He}$ into ${}^4\text{He}$ and two neutrons by²⁰

$$\sigma_\gamma^{\text{E1}}(E_\gamma) = \frac{16\pi^3}{9} \frac{E_\gamma}{\hbar c} \frac{dB^{\text{E1}}}{dE_\gamma}, \quad (10)$$

where $E_\gamma = E + S_{2n}$ is the photon energy, which can be written as the sum of the two-neutron energy $E = E_1 + E_2$ and the two-neutron separation energy, $S_{2n} \equiv Q_{12}$. We refer the reader to the work of Danilin *et al*¹⁹ about details of the calculation.

Table 5: Fit-parameters a_i for the reaction rate $N_A^2 \langle 2n^4\text{He} \rangle$ calculated in our ${}^6\text{He} \equiv \alpha+n+n$ approach and from the E1-strength obtained in the work of Danilin.

	$\alpha + n + n$		Danilin <i>et al</i>	
	$0.1 \leq T_9 < 1$	$1 \leq T_9 \leq 3$	$0.1 \leq T_9 < 1$	$1 \leq T_9 \leq 3$
a_0	2.75605E - 16	-4.31090E - 11	-4.94120E - 11	2.27413E - 10
a_1	2.88001E - 12	1.95298E - 10	2.86628E - 09	-2.40420E - 10
a_2	4.41655E - 12	-3.29920E - 10	-4.90850E - 08	5.64850E - 09
a_3	-1.35270E - 12	2.77932E - 10	3.46189E - 07	-2.23220E - 09
a_4	1.68846E - 11	-1.02300E - 10	-1.08100E - 06	3.69620E - 10
a_5	-2.45810E - 11	1.80468E - 11	1.87337E - 06	-2.62480E - 11
a_6	1.41336E - 11	-1.26680E - 12	-1.85880E - 06	4.500240E - 13
a_7	2.08214E - 11	—	9.89903E - 07	—
a_8	-2.67550E - 11	—	-2.19670E - 07	—
a_9	8.22640E - 12	—	—	—

In Fig. 1 the reaction rates $N_A^2 \langle 2n^4\text{He} \rangle$ for the formation of ${}^6\text{He}$ calculated in the $\alpha+n+n$ approach and deduced from the E1-strength function of Danilin *et al*^{19,21} are depicted. Furthermore, the results obtained in previous studies of Fowler *et al*²² and Görres *et al*²³ are shown (we have used the constructive rate of that publication). Our results obtained in the $\alpha+n+n$ approach show a relatively good agreement with those obtained by Görres²³, who also used a two-step model with a simple direct capture calculation for the second step. Below $T_9 = 0.8$ no data for the rate $N_A^2 \langle 2n^4\text{He} \rangle$ are available in the work of Görres *et al*²³. Our calculations are larger by more than three orders of magnitude compared to the calculations of Fowler *et al*²². This is due to the fact that in the work of Fowler it was assumed that the second step (${}^5\text{He} + n \rightarrow {}^6\text{He} + \gamma$) proceeds via a resonant state in ${}^6\text{He}$ near the threshold followed by the emission of E2-radiation, which is much more less likely to take place than the nonresonant E1 direct capture of a neutron into the ground state of ${}^6\text{He}$.

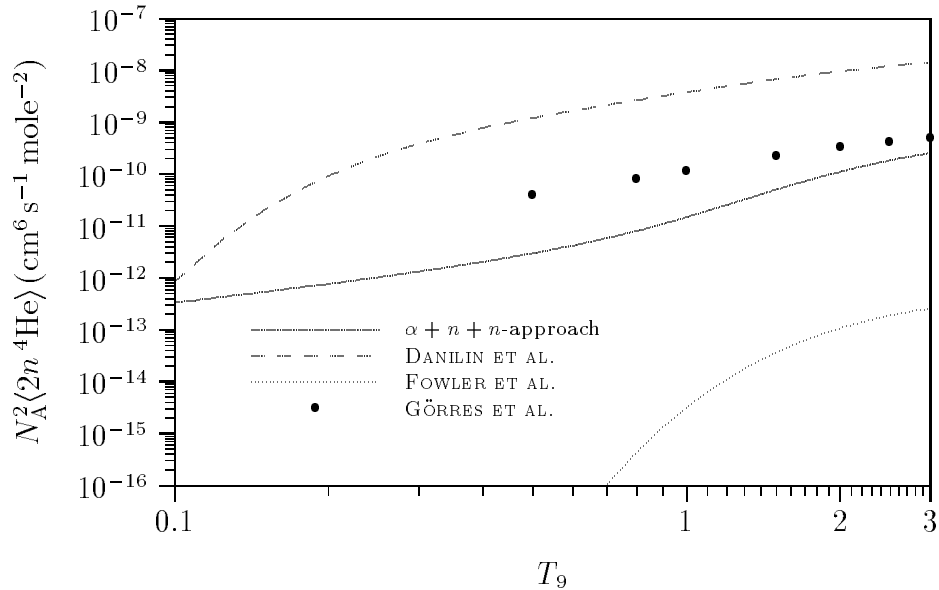


Figure 1: Comparison of different rates $N_A^2 \langle 2n^4\text{He} \rangle$ for the formation of ^6Li .

The most remarkable fact is that the rate deduced from the E1-strength function of Danilin *et al*^{19,21} is for $T_9 > 0.2$ approximately two orders of magnitude larger than our results obtained in the $\alpha+n+n$ approach. A possible explanation of this enhancement is that in our $\alpha+n+n$ calculations only the sequential process is taken into account whereas in the calculations of Danilin *et al*¹⁹ the simultaneous decay ($^6\text{He} + \gamma \rightarrow ^4\text{He} + n + n$) is also included. Furthermore, it is not possible to describe the threshold behaviour of the cross section $\sigma_\gamma^{\text{E1}}$ (with the help of the reciprocity theorem) very well for broad resonances in a two-step model.

4 Summary and Discussion

We calculated reaction rates for one-neutron capture on ^{15}N and ^{16}N and two-neutron capture on ^4He . The rate for $^{15}\text{N}(n,\gamma)^{16}\text{N}$ is

in good agreement with both previous calculations and experimental data. In general the neutron capture rates on stable targets are quite well known. The reaction $^{16}\text{N}(n,\gamma)^{17}\text{N}$ is an example of a capture reaction on an unstable target. We find a considerable enhancement to a previous calculation. This enhancement can be also be observed at various other capture reactions on unstable targets in this mass range²⁴. This fact could influence the reaction path both in the nucleosynthesis of Inhomogeneous Big Bang Models and in the alpha-rich freeze-out of type II supernovae.

For the reaction $^4\text{He}(2n,\gamma)^6\text{He}$ we also find a strong enhancement of the rate compared to the calculation of Fowler²². But from the calculation of the inverse photodisintegration rate by Danilin *et al*¹⁹ we deduce an even much higher rate. This is probably due to the fact that this rate also includes the simultaneous capture of two neutrons. But even with this enhanced rate the path via $^4\text{He}(2n,\gamma)^6\text{He}$ is dominated by the reaction $^4\text{He}(\alpha n,\gamma)^9\text{Be}$ at conditions typical for the alpha-rich freeze-out. This is due to the effective destruction of ^6He through photodisintegration.

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